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Inverse and expansion problems with boundary conditions rationally dependent on the eigenparameter. (English summary)

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The following boundary value problem is considered:

$$ly := -y'' + q(x)y = \lambda y, \quad x \in [0,1], \tag{1}$$

$$(\sin\beta)y'(0) - (\cos\beta)y(0) = 0, \tag{2}$$

$$y'(1) - r(\lambda)y(1) = 0. \tag{3}$$

Here, $\lambda = \rho^2$ is the spectral parameter, $0 \le \beta < \pi$, q is a real-valued function and

$$r(\lambda) = a\lambda + b - \sum_{j=1}^{N} \frac{b_j}{\lambda - c_j},$$

where $a \ge 0$, $b \in \mathbb{R}$, $b_j > 0$, $N \ge 1$ and $c_1 < c_2 < \cdots < c_N$.

Using the asymptotic formulas obtained in [S. Goktas, N. B. Kerimov and E. A. Maris, J. Korean Math. Soc. **54** (2017), no. 4, 1175–1187; MR3668863], the authors consider the inverse problem associated with the boundary value problem (1)–(3). The potential function, which is assumed to be a continuously differentiable function on $[0,1]$, is uniquely determined with the knowledge of the eigenvalues and the corresponding eigenfunctions. Moreover, an expansion formula with respect to the eigenfunctions is obtained for the boundary value problem consisting of (1), (2) with $\beta \in (0,\pi)$, and the boundary condition

$$y'(1) - \frac{b}{\lambda - c}y(1) = 0,$$

where $b > 0$, $c \in \mathbb{R}$, and $\lambda = \rho^2$.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.