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Inverse problem for quasi-periodic differential pencils with jump conditions inside the interval. (English summary)
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In this paper the author studies the boundary value problem B, of the form

$$(1)\qquad y''+\left(\rho^2+\rho p(x)+q(x)\right)y=0,\quad x\in[0,T],$$
$$y(0)=\alpha y(T),\quad y'(0)-(i\rho h'+h)\,y(0)=\beta y'(T),$$
$$(2)\qquad y(b_j+0)=\gamma_j y(b_j-0),\quad y'(b_j+0)=\gamma_j^{-1}y(b_j-0)+(i\rho\eta_j'+\eta_j)y(b_j-0),$$

where $j=1,\dots,N-1$, $0=b_0<b_1<\dots<b_{N-1}<b_N=T$, ρ is the spectral parameter, $p\in AC([0,T];\mathbb{C})$ and $q\in L((0,T);\mathbb{C})$ are complex-valued functions, and $h',h,\alpha,\beta,\gamma_j,\eta_j',\eta_j$ are complex numbers satisfying $\alpha\beta\gamma_j\neq 0$.

In Section 2, he defines the special solutions $S(x,\rho)$ and $C(x,\rho)$ of the differential equation (1) which satisfy the jump conditions (2) and the initial conditions

$$S(0,\rho)=C'(0,\rho)=0,\quad S'(0,\rho)=C(0,\rho)=1.$$

The characteristic function is given and it is said that the eigenvalues $\{\rho_n\}_{n\in\mathbf{Z}}$ coincide with the zeros of this characteristic function.

With the help of the notations $\varphi(x,\rho)=C(x,\rho)+(i\rho h'+h)S(x,\rho)$, $d(\rho)=S(T,\rho)$ and $d_1=C(T,\rho)$ the function $a(\rho)$ is defined as

$$a(\rho)=\alpha\varphi(T,\rho)+\beta S'(T,\rho)-(1+\alpha\beta).$$

Then, denoting $Q(\rho)=\alpha\varphi(T,\rho)-\beta S'(T,\rho)$, the author sets

$$w_n=\begin{cases}0, & Q(\nu_n)=0,\\ +1, & Q(\nu_n)\neq 0,\ \arg Q(\nu_n)\in[0,\pi),\\ -1, & Q(\nu_n)\neq 0,\ \arg Q(\nu_n)\in[\pi,2\pi),\end{cases}$$

for $n\in I:=\{n\in\Lambda:\nu_{n-1}\neq\nu_n\}$, where $\Lambda:=\mathbf{Z}\setminus\{0\}$, and $\{\nu_n\}_{n\in\Lambda}$ are the eigenvalues of the boundary value problem for equation (1) with jump conditions (2) and the boundary conditions $y(0)=y(T)=0$.

After that, the sequence which is called the Ω-sequence for B is given: $\Omega=\{w_n\}_{n\in I}\cup\{w_{n\nu}\}_{n\in I_0}$, $\nu=1,\dots,m_n-1$, where $w_{n\nu}:=d_1^{(\nu)}(\nu_n)$, $\nu=0,\dots,m_n-1$, $I_0=\{n\in I':w_n=0\}$, $I'=\{n\in I:m_n>1\}$, and m_n is the multiplicity of ν_n.

Inverse Problem I. Given $a(\rho)$, $d(\rho)$ and Ω, construct B.

Section 3 is devoted to studying the properties of the spectral characteristics. To do this, first the Weyl-type function

$$M(\rho)=-\frac{d_1(\rho)}{d(\rho)}$$

is given. After proving a necessary lemma and giving some necessary notations the Weyl sequence is defined as

$$M_n=-\frac{d_1(\nu_n)}{d'(\nu_n)},\quad n\in\Lambda.$$

The data $D=\{\nu_n,M_n\}_{n\in\Lambda}$ are called spectral data and it is noted that the specification of the spectral data D uniquely determines the Weyl-type function $M(\rho)$. Section 3 ends by proving a uniqueness theorem for the inverse problem-0 which is: given the spectral data $D = \{\nu_n, M_n\}_{n\in\Lambda}$ and γ_j, $j = 1, \ldots, N-1$ construct $p(x)$, $q(x)$, $x \in (0,T)$, ν_j', ν_j, $j = 1, \ldots, N-1$.

In Section 4, the solution of the Inverse Problem I is constructed and the algorithm of this construction is given. *F. Ayca Cetinkaya*

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