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**Defect basis property of a system of root functions of a Sturm-Liouville problem with spectral parameter in the boundary conditions. (English summary)**

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In this paper the authors consider the following Sturm-Liouville problem with spectral parameter in the boundary conditions:

$$(1) \quad y''(x) + q(x)y(x) = \lambda y(x), \quad x \in (0, 1),$$

$$(2) \quad (a_0\lambda + b_0)y(0) = (c_0\lambda + d_0)y'(0),$$

$$(3) \quad (a_1\lambda + b_1)y(1) = (c_1\lambda + d_1)y'(1),$$

where  $\lambda$  is the spectral parameter,  $q(x)$  is a real continuous function on  $[0, 1]$ , and  $a_i, b_i, c_i$  and  $d_i, i = 0, 1$ , are real constants satisfying

$$\sigma_0 = a_0d_0 - b_0c_0 < 0 \quad \text{and} \quad \sigma_1 = a_1d_1 - b_1c_1 < 0.$$

Note that the signs of the values  $\sigma_0$  and  $\sigma_1$  play an important role. If  $\sigma_0 < 0$  and  $\sigma_1 > 0$ , problem (1)-(3) can be treated as a spectral problem for a self-adjoint operator in the Hilbert space  $H = L_2(0, 1) \oplus \mathbf{C}$ .

The main objective of the paper is to study the basis properties of systems of root functions of the spectral problem (1)-(3) in the space  $L_p(0, 1)$  ( $1 < p < +\infty$ ). In Section 2, the structure of the root subspaces and the oscillation properties of eigenfunctions of problem (1)-(3) are studied. In Section 3, the operator interpretation of problem (1)-(3) is given and the construction of a system biorthogonal to the system of root vectors of the corresponding operator is examined. In Section 4, a necessary and sufficient condition as well as sufficient conditions for the basis property of the systems of root functions of problem (1)-(3) for  $\sigma_0 < 0$  and  $\sigma_1 < 0$  is given. *F. Ayca Cetinkaya*

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*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*