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The inverse spectral problem for the perturbed harmonic oscillator on the entire axis. (English summary)
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In [Comm. Math. Phys. **82** (1981/82), no. 4, 471–495; MR0641910] H. P. McKean Jr. and E. Trubowitz considered the problem of reconstructing the perturbed oscillator

$$T = \widehat{T} + q(x), \quad \widehat{T} = -\frac{d}{dx} + x^2$$

on the real line They gave an algorithm for the reconstruction of $q(x)$ from norming constants for the class of real infinitely differentiable potentials, vanishing rapidly at $\pm\infty$, for fixed eigenvalues $\lambda_n(q) = \lambda_n(0)$ for all n and "norming constants" $\to 0$ rapidly as $n \to \infty$.

Later, in [Dokl. Akad. Nauk SSSR (N.S.) **105** (1955), 637–640; MR0080735] M. G. Kreĭn re-proved some results of [op. cit.] without an exact definition of the class of potentials. It was also noted there that the perturbation potentials may be constructed by the standard procedure of the method of transformation operator. However, the rationale for some heuristic considerations of [H. P. McKean Jr. and E. Trubowitz, op. cit.] will require the construction of a transformation operator with a condition at infinity.

In this article, Bagirova and Khanmamedov consider the perturbed oscillator T, generated on $L_2(-\infty, \infty)$ by the anharmonic equation

$$-y'' + x^2 y + q(x)y = \lambda y, \quad -\infty < x < \infty, \quad \lambda \in \mathbb{C},$$

where the real potential $q(x)$ satisfies the conditions

$$q(x) \in C^{(1)}(-\infty, \infty), \quad \int_{-\infty}^{\infty} \left|x^j q(x)\right| dx < \infty, \quad j = 0, 1, 2.$$

They construct transformation operators with a condition at infinity for the perturbed oscillator T. Furthermore, transformation operators are used to solve the inverse spectral problem for the perturbed harmonic oscillator T having the same spectrum as the harmonic oscillator $\widehat{T}$. The main equations of the inverse problem are obtained, and their unique solvability is proved. *F. Ayca Cetinkaya*