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**The modified Parseval equality of Sturm-Liouville problems with coupled boundary condition. (English summary)**

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In this paper, the authors consider the differential equation

$$ly := -y'' + q(x)y = \lambda y, \quad x \in J = [-1,0) \cup (0,1]\,,$$

with the boundary conditions

$$AY(-1) + Y(1) = 0, \quad Y(\pm 1) = \begin{bmatrix} y(\pm 1) \\ y'(\pm 1) \end{bmatrix}$$

and the transmission condition

$$KY(0-) + Y(0+) = 0, \quad Y(0\pm) = \begin{bmatrix} y(0\pm) \\ y'(0\pm), \end{bmatrix}$$

where $\lambda$ is a complex eigenparameter, $A$ and $K$ are $2 \times 2$ matrices of the form

$$A = e^{i\gamma} \begin{bmatrix} \alpha_1 & \alpha_2 \\ \alpha_3 & \alpha_4 \end{bmatrix} \quad \text{and} \quad K = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix},$$

in which $-\pi \le \gamma \le \pi$, $\alpha_1\alpha_4 - \alpha_2\alpha_3 > 0$ and $k_{11}k_{22} - k_{12}k_{21} > 0$. The matrices $(A, -I)$ and $(K, -I)$ have full rank, where $I$ is the $2 \times 2$ identity matrix, and $q \in L(J, \mathbf{R})$.

The condition for $\lambda$ to be an eigenvalue of the above boundary value problem and the condition for eigenvalues of this problem to be countably infinite are given. The Green's function is constructed. The eigenfunction expansion for the Green's function is derived and a modified Parseval equality is established by using the eigenfunction expansion. *F. Ayca Cetinkaya*

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*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*