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Uniform, on the entire axis, convergence of the spectral expansion for Schrödinger operator with a potential-distribution. (English summary)
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In this paper, the author proves a uniform estimate for the increment of the spectral function $\theta(\lambda; x, y)$ at $x = y$ for the one-dimensional self-adjoint Schrödinger operator $\mathcal{A}$ generated in the space $L_2(\mathbb{R})$ by the differential operation

$$L = -\frac{d^2}{dx^2} + q(x), \quad x \in \mathbb{R}, \tag{1}$$

with a real potential-distribution $q(x)$ from the space $W^{-1}_{2,\mathrm{loc}}(\mathbb{R})$, that is, a space adjoint to the Sobolev space $\overset{0}{W}{}^1_2(\mathbb{R})$ of functions compactly supported on $\mathbb{R}$.

Let $\{E_\lambda\}_{\lambda\ge 1}$ be the family of spectral projections (resolution of the identity) that corresponds to the operator $\mathcal{A}$. It follows from the general theory that the spectral expansion $\sigma_\lambda(x, f) \equiv E_\lambda f(x)$ of any function $f \in L_2(\mathbb{R})$ converges to $f(x)$ in the metric of this space.

The paper aims at deriving conditions on the function $f(x)$ under which the relevant spectral expansion converges in a metric that is uniform on the entire axis $\mathbb{R}$. The key result is given by a uniform, with respect to $x \in \mathbb{R}$, estimate for the increment of the kernel of an integral operator that defines the spectral projection E_λ (the so-called spectral function $\theta(\lambda; x, y)$) for $x = y$.

The classical form estimate for the increment of the spectral function

$$\sup_{x\in\mathbb{R}} |\theta((\mu+1)^2; x, x) - \theta(\mu^2; x, x)| = O(1), \tag{2}$$

(the constant in $O(1)$ is independent of μ) is proved in the present paper for the self-adjoint operator $\mathcal{A}$ that is defined by the differential operation (1) with an arbitrary potential from the class $W^{-1}_{2,\mathrm{unif}}(\mathbb{R})$.

As a corollary to the estimate (2), it is shown that the spectral expansion $\sigma_\lambda(x, f)$ absolutely and uniformly, on the entire axis $\mathbb{R}$, converges to $f(x)$ if the expanded function $f(x)$ belongs to the domain $D(\mathcal{A}^\alpha)$ of the operator $\mathcal{A}^\alpha$ with exponent $\alpha > 1/4$. In addition, an estimate for the rate of this convergence in the form

$$\sup_{x\in\mathbb{R}} |\sigma_\lambda(x, f) - f(x)| = o(\lambda^{(1/4)-\alpha}) \tag{3}$$

is derived. It is noted in the paper that the estimate (3) completely coincides with the convergence rate estimate in the regular case [S. A. Denisov, Differ. Uravn. **33** (1997), no. 6, 754–761, 861; MR1615055]. *F. Ayca Cetinkaya*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.