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Metrics on a closed surface of genus two which maximize the first eigenvalue of the Laplacian. (English, French summaries)
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Let M be a closed surface, that is, a compact surface without boundary. M is assumed to be orientable. For a Riemannian metric ds^2 on M, let

$$\Lambda(ds^2) := \lambda_1(ds^2) \cdot \mathrm{Area}(ds^2),$$

where $\lambda_1(ds^2)$ is the first positive eigenvalue of the Laplacian and $\mathrm{Area}(ds^2)$ is the area of M, both with respect to ds^2.

In this paper, the authors settle in the affirmative the Jakobson-Levitin-Nadirashvili-Nigam-Polterovich conjecture [D. Jakobson et al., Int. Math. Res. Not. **2005**, no. 63, 3967–3985; MR2202582], which is as follows:

Conjecture: $\lambda_1(ds^2_B) = 2$ should hold. Therefore, $\Lambda(ds^2_B) = 16\pi$.

To settle this conjecture in the affirmative, the authors state that a certain singular metric on the Bolza surface, with area normalized, should maximize the first eigenvalue of the Laplacian. *F. Ayca Cetinkaya*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.