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Trace formula for a Sturm-Liouville operator with a δ'-interaction point. (English summary)

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In the present work, the authors consider the eigenvalue problem

$$-y''(x)+q(x)y(x)=\lambda y(x),\quad x\in(0,\pi/2)\cup(\pi/2,\pi), \tag{1}$$

with the boundary conditions

$$y'(0)-hy(0)=0,\quad y(\pi)=0 \tag{2}$$

and the jump conditions

$$y'(\pi/2+0)=y'(\pi/2-0)\equiv y'(\pi/2),\quad y(\pi/2+0)-y(\pi/2-0)=\alpha y'(\pi/2), \tag{3}$$

at the point $x=\pi/2$, where h and $\alpha\neq 0$ are given real numbers, λ is the spectral parameter, and $q\in W_1^1(0,\pi)$ is a real function.

Equation (1) with the discontinuity conditions (3) can be reduced to the equation

$$-y''(x)+(\alpha\delta'(x-\pi/2)+q(x))y(x)=\lambda y(x),\quad x\in(0,\pi), \tag{4}$$

where $\delta'(x)$ is the derivative of the Dirac delta function.

In this paper, the authors obtain a first-order regularized trace formula for the Sturm-Liouville operator L generated in the space $L_2(0,\pi)$ by the differential expression

$$\ell y:=-y''+q(x)y$$

with dense domain

$$\mathcal{D}(L):=\Big\{y\in W_2^2[(0,\pi/2)\cup(\pi/2,\pi)]\cap L_2(0,\pi): y'(0)-hy(0)=0,\ y(\pi)=0,$$
$$y'(\pi/2+0)=y'(\pi/2-0)\equiv y'(\pi/2),\ y(\pi/2+0)-y(\pi/2-0)=\alpha y'(\pi/2)\Big\}.$$

They prove that the operator L has countably many eigenvalues λ_n, $n=1,2,\dots$, and show that these eigenvalues have the following asymptotics as $n\to\infty$:

$$\lambda_n=n^2+\frac{2}{\pi}\left(\omega_1+(-1)^{n-1}\omega_2\right)+\frac{\sigma_n}{n},\quad \{\sigma_n\}\in l_2. \tag{5}$$

The authors mention that by the *first-order regularized trace* for the boundary value problem (1)–(3) (and, hence, also for problem (4),(2)) they understand the sum of the series

$$S_\lambda:=\sum_{n=1}^{\infty}\left(\lambda_n-n^2-\frac{2}{\pi}\left(\omega_1+(-1)^{n-1}\omega_2\right)\right). \tag{6}$$

The asymptotics (5) shows that the series S_λ converges. The aim of the paper is to find the sum of the series (6). *F. Ayca Cetinkaya*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.