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Inverse problems for Dirac operator with the potential known on an interior subinterval. (English summary)

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Inverse spectral problems for the Dirac operator $L := L(Q(x); \alpha, \beta)$ generated by a differential expression of the form

$$ly := By' - Q(x)y = \lambda y, \quad 0 < x < 1,$$

with

$$B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad Q(x) = \begin{pmatrix} p(x) & 0 \\ 0 & q(x) \end{pmatrix}, \quad y(x) = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix}$$

subject to the boundary conditions

$$U(y) := y_1(0)\cos\alpha + y_2(0)\sin\alpha = 0,$$
$$V(y) := y_1(1)\cos\beta + y_2(1)\sin\beta = 0,$$

where $0 \le \alpha, \beta < \pi$, are considered. Here, λ is the spectral parameter, and $p, q \in C[0,1]$ are real-valued functions.

It is proved that both the potential on the whole interval and the boundary conditions are uniquely determined by the potential on an interior subinterval $[a_1, a_2] \subset [0,1]$ including the midpoint $1/2 \in [a_1, a_2]$, together with partial eigenvalues and partial interior spectral data.

The technique which is used to obtain the uniqueness result is based on the method discussed in [M. Horváth, Trans. Amer. Math. Soc. **353** (2001), no. 10, 4155–4171; MR1837225]. *F. Ayca Cetinkaya*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.